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Contents

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POWER AND STAGNATION

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I. MAIN PRODUCTIONS

(A) Utility Economy (model usually assumed in orthodox economics) is an economic system in which all units (firms and family economies) act in purely economic ways, getting commodity X merely by delivering other commodity Y (by pure exchange).

Power economy is an economic system in which all units act in powerusing ways, getting commodity X by delivering Y (that is, in the form of exchange) and at the same time by exercising power.

(B) Böhm's theory which asserts the inefficacy of power presupposes utility economy. Orthodox economics is a system of economic laws which prevail in utility economy (involving, especially, perfect wage flexibility).

Keynesian economics presupposes the peculiar supply function of labour which again must presuppose the power economy based on power relations. Since we accept the model of power economy, the Böhm's theory of inefficacy of power cannot be held valid. For example, if the required wage be raised, unemployment will increase in volume, without suppressing wage by competition.

(C) Marginal productivity theory invalid. Marginal productivity theory states that the price of a factor is determined by the marginal product of that factor (for example, labour) in the case of full utilization. In the past eighty years, full employment has been rare and exceptional, unemployment being usual. Wage tends to be raised higher than marginal product of labour (in the case of full employment), by power resistance of labourers, unionized or not.

(D) Possible forms of supply functions of labour. (a) Parallel to y axis (Böhm-Bawerkian labour supply), (b) parallel to x axis) Keynesian labour supply), (c) increasing supply of labour inrelation to wage (Walrasian and Marshallian). (a) is not compatible with everylasting unemployment, (c) cannot explain involuntary unemployment, (b) presupposes resistances by power of which Keynes is not aware himself.

(E) Wage rigidity at a short period and secular trend of continuous rising in wage can be explained neither by labour monopoly nor by rising marginal productivity. Not by labour monopoly (through union), because the said rigidity is rather general, being perceived at the time when unionization was weak and irrespective of the degree of unionization. Not by rising marginal productivity, because the real wage rises when unemployment is increasing.

(F) Involuntary unemployment as a product of power-influenced supply of labour. Apparently involuntary unemployment is caused by insufficiency of effective demands and through "power-influenced" (Keynesian) labour supply. Which of the two is the true cause? If the orthodox (Walrasian) theory of supply function of labour is valid, involuntary unemployment must vanish. Insufficiency of effective demands is brought about by the power supply function of labour which hinders the downward movement of required wage.

(G) Degeneration of capitalism as the true aspect of the so-called stagnation or maturity. The stage of utility economy in the growth of capitalism has passed away, and it has plunged into the stage of power economy which represents the degeneration of capitalism. Social atomization, consequently the emancipation of individuals is the trend in the long run, especially achieved in European culture, which has been developed in and after the World Wars. The tendency has brought about two results (a) higher wage rates compared with marginal product, (b) restricted economic circles, that is, the reduation of the scope for international economy (segregation of communistic spheres, industrial progress of backward areas). In this case I put stress on the former point. High wage by power resistance of labouring strata has brought about the world-wide tendency of the increase in unemployment, con-

sequently the so-called maturity, which needs everywhere a growing autonomous investment as a remedy. Depression by the supremacy of labour (labourism) is the kernel of maturity.

This section is the last part of my paper, read before an assembly (May 7, 1955) in the Kyoto American Seminar (directed by Professor Fritz Machlup). Professor Machlup gave me then orally valuable criticisms and suggessions concerning many points for which I express my deep gratitude. The following sections consist of several comments, written to make clear my ways of thinking on chief items.

II. COMMENT ON A SECTION

I have pursued in section (E) of my above propositions the following way of reasoning. Wage rigidity and secular trend of continuous rising in wage can neither be explained by labour monopoly nor by rising marginal productivity. Not by labour monopoly (through union) because that rigidity is rather general, being perceived even at the time when unionization was weak, and seen irrespective of the extent of unionization. Not by marginal productivity, because the real wage rises when unemployment is increasing.

The purport of the passage may be divided into two parts.

(a) The rigidity of money wage (at a short period) is usually be explained by the labour monopoly through unionization but the wage rigidity mentioned above is so general in time and space that it prevails already when and where labour is not unionized or it is unionized yet to a small extent. Moreover it must be considered that unionization itself is a product of the emancipation or the resistance by power of the labouring class so that the so-called monopoly of labour cannot be deemed simply as the exchange phenomenon, since it usually contains some kinds of political struggles. But the stress must be put on the following argument.

Usually the continuous tendency of wage-raising is explained as a result of the changing or increasing marginal productivity of labour. First it presupposes that the ratio of capital to labour increases incessantly with the process of capitalistic expansion, so that the marginal

productivity of labour increases, and that of capital decreases if the same method of production is used continually. Secondly, the method is always changing and the result of it does not show a definite direction at any given point. It may save labour in relation to capital, that is, capital is substituted for labour and this tends to decrease the marginal productivity of labour, when all labours are used. But at the same time it is usual that it increases the whole products of society so that both the marginal productivity of labour and that of capital may rise, though at different tempo.

These features do not permit a simple generalization, all depending upon concrete forms of production function or characteristics of every technique. As a resultant effect of those two tendencies, increase of per head capital and change of production function, the marginal productivity of labour cannot take a definite direction, that is, either upward or downward. At least we are not able to say anything about its direction without having more concrete material about production functions. But roughly speaking, the former tends to increase the marginal productivity of labour, while the latter may decrease it, and the resultant effect may be an upward tendency. Thus the supporters of the marginal productivity theory may explain the rising tendency of wage as an effect of the change in productivity. That is probable but not necessary ; if we say it is necessary it is obvious that the reasoning runs in a circle. I declare I do not believe in the theory of marginal productivity without hesitation. I entertain many doubts about its general validity, but this is not a proper occasion for dwelling upon the subject. Only one thing must be made clear here.

The object of marginal productivity theories lies in the explanation of factor prices ; to confine myself to wage, the theory asserts that wage rate is determined by the productivity of marginal unit, when labour is used without rest, that is, labour is in the state of full employment ; to repeat the point, wage is determined by the marginal productivity under full utilization. If not yet fully utilized, the wage will be cut down by the competition of the unemployed, until it equals the marginal productivity. It does not matter whether the supply curve of labour is of the Böhm-Bawerkian form or of the Marshallian form. This is

the presupposition of the following argument.

The supporters of the marginal productivity theory of wage consider that the secular rising tendency of wage is due to the incessant increase in the marginal productivity of labour. But this is improbable because (1) it is difficult or rather impossible to demonstrate that the marginal productivity of labour is always increasing with the expansion of production, without going into a circular reasoning, the exact calculation of labour productivity in each period being very difficult, (2) since there usually exists a greater or smaller volume of unemployment the marginal productivity of labour must be lower than the current wage which is bounded to go down by cutting through competition at successive periods. This means that the current wage is always higher than marginal productivity which disproves the validity of the presumed theory.

Thus to be a valid wage theory, it must be able to demonstrate the necessity of the coexistance or combination of unemployment and wage higher than marginal productivity. The only one which fulfils such requirement we can find in the power theory. Wage being determined by power at the level higher than marginal productivity under the prevailing situation which makes labour suppliers require higher and higher wage rate owing to the pace of increasing power resistance. Higher wage by power resistance necessarily causes a certain volume of unemployment. One may be convinced that any other theory of wage can not explain the concomitant existance of unemployment and higher wage. Now I have no intention of entering into the difficult problem of wage policies. The shortest way to realize full employment and equilibrium wage seems to cut down money wage level generally, but of course the matter is not simple concerning which a long series[1] of

[1] The following objection must be expected. How is it possible to cut down real wage rate through reduction of money wage generally? It is often concluded that the general cut in money wage rate must cause the proportional fall in every price so that the situation of real economy will not change, real wage rate as well as employment remaining at the previous level. That conclusion is insisted upon by the Keynesians but their standpoint is not consistent. The idea of involuntary unemployment denies the homogeneity of demand and supply function and the idea of proportional price change presupposes the very homogeneity. Further, it seems to me, the proportional fall of every price is possible only under some special money policy which may not be generally expected so that the prestige of money or the inertia of money prices and like phenomena will cause the reduction of real wages. If the reduction of renal wage through money wage cut is impossible, likewise the raising of real wage must be impossible so that the real wage at present may be unchangeable so far as the change of money wage rate is the only means of increasing or decreasing real wages.

controversies has been continued without attaining any unified conclusion.

The above analysis assumes that the way of thinking of marginal productivity is acceptable if we presuppose the model of utility economy, that is, pure exchange economy, but this assumption is not necessarily right, since the theory is only applicable so far as the principle of substitution prevails among productive goods, which means it is inapplicable so far as the principle of complementariness reigns. In the latter case, the calculation of marginal productivity of definite positive value will be impossible. Perhaps the concept of marginal productivity must give way to the concept of marginal net productivity or that of marginal residual productivity since in the capitalistic economy, no factor alone can be increased without simultaneous increase of capital so that partial differentials indicating marginal productivity have no meaning.

III. MONOPSONY AND MARX

Concerning the marginal productivity theory of wage there remain unsolved points, considering which we attain to further criticisms, but now we turn to another important wage theory which we must not neglect to analyze.

As I have often stated elsewhere, trials to apply a kind of monopoly concept in explanation of wages were made by some authorities.

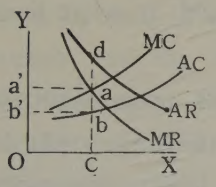
Let us first take up the two representative kinds of monopoly theory of wage. One of them is the monopsony theory of wage which tends to stress the facts of exploitation, the other is the union monopoly theory of wage which now prevails in labour economics.

First I shall concentrate my attention on the monopsonistic trend among monopoly theories of wage and analyze it to see to what extent it succeeds in explaning the wage phenomena without introducing power factor. Adopting the idea of the exploitation by the capitalist in the Marxian sense, some authorities hold the conviction that monopsonists pay as much as average cost, therefore less than marginal cost so that the difference between them constitutes the capitalist's exploitation.

Monopsony wage is shown in Fig. I by height *bc*. *ac–bc*, i.e. *ab*

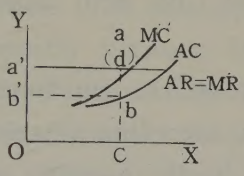
shows monopsonistic profit per labour which is valid also for Fig. II, representing the case of competition in products market and of monopsony in factor market. If we accept the idea of involuntary unemployment as all Keynesians do, the therein assumed supply curve of labour must be *aa'* in Fig. IV, which is transplanted as *aa'* in Fig. III, then no monopsony can exist because *mc* equals *ac* so that monopsonistic profit per labour *ab* is reduced to zero. The necessary consequence is that the monopsony theory of wages and the concept of involuntary unemployment cannot be consistent with each other; they are incompatible and contradictory.

Fig. I



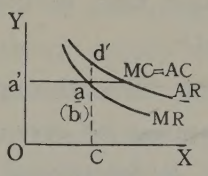
A. Monopoly of produts, monopsony of factors.

Fig. II



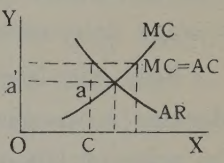
B. Competition of products, monopsony of labour.

Fig. III



C. Social demand curve, union supply of labour.

Fig. IV



D. Involuntary unemployment with no monopsony profit.

MC ··· marginal cost
AC ··· average cost
MR ··· marginal revenue
AR ··· average revenue

bc ··· monopsony wage (per labour)
ab ··· monopsony profit (")
ad ··· monopoly profit (")

From where has the contradiction come? I think that each theory has its proper assumption. While the theory of exploitation by monopsony contains in itself the idea of the labour market of imperfect demand monopoly and the weak position of labour which endures the burden of exploitation, the theory of involuntary unemployment is based on the data that the form of labour market is competition and the supply price of labour is not determined by disutility of labour but by power-resistance of labour, unionized or not. Thus the above-mentioned incompatibility is not a theoretical one but is a mere reflex of the difference of presupposed situations. But which of the two different data is nearer to the present economic reality? Monopsony market of labour seems to be the relic of the previous underdeveloped stage of capitalism since the immobility of labour may decrease more and more, except in

the case of gigantic firms, the products of which hold a position of supply monopoly but which encounter the counterbalancing power of supply monopoly of union. Anyhow the further the capitalism develops, the stronger the power resistance of labour will become so that the involuntary unemployment may perhaps increase incessantly. In this sense, the labour supply influenced by power will become dominant in future, which assumption leads us to further analysis of its characteristics.

As a preliminary step to the analysis, it will be useful and important to give some consideration to the Marxian theory of wage. In some sense it is the pivot of Marxian economics, from which almost all his conclusions must have been derived. By what cause and at what level is wage determined?

First his labour value theory as the fundamental proposition of Marxian economics must be given, according to which the wage, that is, the price of labour, is determined by the quantity of labour necessary to the production. Wage being the price of labour force, it must be determined at the same level as the cost of the reproductions of labour force which is nothing else than the usual standard of living. But how has the standard of living been formed in the past? The only possible answer is that history has made it, but in reality this can be called no theoretical answer.

Next the level of wage is not stated by Marx in a definite way. In some cases real wage is thought to be constant which will make the rate of exploitation always increase. In other cases wage in labour value is held to remain unchanged while technique is progressing, so that real wage must be increasing. Real wage is further stated to be always decreasing as the effect of the increase of unemployment, without saying anything about the fluctuation of wage owing to a short duration of demand and supply relations.

In short, there are many Marxian theories of wage level whcih contradict with each other. But the central or fundamental view must be the Verelendungstheorie, the theory of increasing misery which must not be understood in the sense of relative poverty which is compatible

with ever increasing real wages.

One thing is clear and evident. According to the Marxian conception, labour produces all national income and only some part of it is paid back to labour but the level of wage cannot be explained by the mere labour value theory, since the reproduction cost of labour force may be fixed at any height. Marx made history decide it, but history means *a*-theory. A new factor must be introduced into economic theories, which stands in contrast to utility or preference, and that will be the power, the validity of which may be proved by theoretical consistencies or achievements.

The same argument may be applied to the concept of involuntary unemployment of Keynes. The possibility of involuntery unemployment depends on the disparity between labour supply curve and labour disutility curve. If the fomer is identical with the latter, there may be only the voluntary unemployment. Since there exists always the disparity between the two, the utility of wage may be higher than disutility of labour.

But what makes the disparity possible or necessary? It must be the factor of power. The will to power makes us strive to attain more power and to show the power gained, power meaning here the capacity for commanding the obedience of others by coercive force, position or dignity. To require such and such a quantity as one's income (as one's wage) means to claim it as the sign or symbol of his respective degree of social power or position apart from the contents of real utilities purchased with it. The rigidity of wage, or the historically established standard of living as required wage and even the supply wage in wage dispute may be inexplicable without introducing the element of power.

There are very wide and various fields of research in wage phenomena which can neither be explained nor understood, such as wage-difference by social strata (e. g. manual labour and white collar, male and female, nationality *A* and *B*, living as members of a community, etc.. Such a hierarchy of wages or wage differences can never be explained by the theory of marginal productivity or that of monopoly theory of various kinds; different wage-rates for work same in kind and

productivity are inexplicable in that way.

IV MARGINAL PRODUCTIVITY THEORY

As the last point of our argument, let us add some remarks on the theoretical form of the marginal productivity theory. We assumed in the above analysis that the theory aims at explaining the level of wages. Of course we know that there is prevalent an idea according to which the theory states only that wage equals marginal productivity, at whichever level wage may be fixed; it would not assert that marginal productivity determines wage. But we believe that, if the concept of marginal productivity does not involve in itself the proposition concerning the determination of factor price, its theoretical significance is not great indeed.

To give some examples, there are the so-called American school of the distribution theory from J. B. Clark to Paul Douglas and the Walrasians including the Casselian direction of the general equilibrium theory to say nothing of other neoclassical scholars; their intention has always been to explain the level of wages by marginal productivity. But we know, the situation has changed.

The premise, which the theory has assumed, that is, the assumption of full employment, is not generally realized so that its actual validity may not be accepted. The concept of marginal productivity remains now as an explanation of demand function of factor, the supply side being taken as it actually is and left unexplained. This new position of economic thinking which I must suppose to be common in the wide circle of scholars, must be taken into consideration and in the light of it I have to reconsider my standpoint.

Anyhow the marginal productivity theory seemed to be the only one living explanation of wage but now that its premises have failed, its validity must be denied.

(1) Though the marginal productivity function be given, wage rate cannot be determined since supply function of labour is indeterminate. The missing link in the chain as the definite supply function must be given, to explain which there remains no other factor than social power.

(2) To deny the marginal productivity theory as an explanation of wage rate only in order to accept the idea of marginal productivity as the principle of factor demand, I am afraid, will mean the loss of wage theory in general, since no positive proposition which will furnish labour supply function with some substantial connotation is not given.

(3) In the present state of economics every concept of labour supply must accept and explain the fact of involuntary unemployment. Without fulfilling the function, no wage theory can claim validity. And I repeat, it will be impossible to explain involuntary unemployment without introducing power element into the concept of labour supply.

(4) In the present situation every wage theory has to explain the everlasting tendency of ascending real wage. Previous theories, such as the living standard theory, old wage fund theory, break down at this point. The marginal productivity theory seemed alone to bear this test. But the premises necessary for the general validity of this theory have never been realized, at least during this century which always denies any explanation through applying it to the actuality. Without considering an active and aggressive attitude on the side of labour supply, the above-mentioned situation cannot be understood.

The element of power is conceived to mean the pressure of trade union in collective bargaining but it is too far from the intention of my report. Labourers have their own respective positions, socially and politically before entering into wage contract, which make them claim their required wage and make employers accept them. It is usual that wage rate is determined without intervention of unions during long before the advent of it, the pressure of which is a mere accident in the process of the wage history. In the last analysis, my intention lies in the thesis that power determines wage rate as the complementary factor against labour demand.

Sometimes it is said that consumers' goods are also determined by power, for example by price regulation just like as wage rate by union pressure, so that there is no distinction between the two prices. But it is not the purport of our power theory ; prices of usual commodities are

determined simply by utility calculation on both partners because such commodities are separated from persons, but labour is not so because it is self-conscious goods like "a fictitious apple crying its price".

REPLY TO PROFESSOR TAKATA

FRITZ MUCHLUP

I. INTRODUCTION

Every year, for the last 28 years, I told the students in my classes, perhaps very wrongly but with sincere conviction, the following story about the marginal productivity theory.

There are many teachers who present what they call the marginal productivity theory as a theory of wages. I believe that this should not be done, because (1) there is not one "marginal productivity theory" but there are several, so that it would be preferable to speak of a "marginal productivity pricniple" which can be used in different theories, and (2) the marginal productivity principle is not used as a "theory of wages", or a theory of factor prices, but only as a theory of demand for factors of production.

(1) The marginal productivity principle in used in at least four different theories:

(a) in the theory of input determination by the individual firm;

(b) in the theory of factor employment by the particular industry;

(c) in the theory of the factor market; and

(d) in the theory of general equilibrium.

In these different theories the m. p. principle is combined with several different sets of assumptions; for example, in the theory of the firm we are not constrained to assume either pure or perfect competition, but may assume any kind of monopoly or oligopoly, monopsony or oligopsony. In the theory of the industry we must assume pure competition among the firms, but there may well be monopoly on the factor supply side; moreover, it is not necessary to assume perfect newcomers' competition (pliopoly in my terminology) but we may assume an industry closed to new entrants. In the theory of the factor market we are confined to more stringent assumptions; we shall have to assume pure insiders' competition as well as perfect newcomers'

competition in the industries which compose the demand for the factor; we are still free to assume a monopolistic supply of the factor. In the general equilibrium theory of the Walrasian form we are limited to assumptions of pure and perfect competition all around. There are some recent attempts to expand general equilibrium theory enough to permit imperfection of competition among the assumptions but this is still controversial among the theorists.

(2) The marginal productivity principle alone can never explain the wage rate, because a demand curve alone can never explain a price. A supply curve must furnish the other blade of the theoretical scissors. In the limiting case of a "perfect market" with "perfect competition" it is sometimes said that, given the supply curve of labor, the marginal productivity curve will determine the wage rate; but with equal justification one could say in this case that, given the marginal productivity curve of labor, the supply curve will determine the wage rate. Both expressions had better be avoided.

(3) We should avoid expressions such as "marginal productivity" rises or falls, or is raised or lowered, because it is never clear whether we mean

(a) marginal productivity at the given volume of employment,
(b) marginal productivity at an adjusted smaller or larger-volume of employment,
(c) marginal productivity at a volume of employment at which the factor supply is fully employed, or
(d) marginal productivity at any volume of employment, that is, the whole curve.

It is usually possible by proper safeguards to make clear which of these four meanings we have in mind.

(4) Since we must distinguish between
(a) the marginal physical product,
(b) the value of the marginal physical product,
(c) the marginal value product (or marginal revenue product), and
(d) the marginal net revenue product,

We should use such language as would avoid confusions between these four concepts of marginal productivity.

II. TERMINOLOGY

I hope you will forgive me for troubling you with this long introduction. I did it in order to explain to you some of my oral comments on your paper. Now it will be easy to see that there is no *real* difference of opinion between us, but only a difference in the meaning we have attached to our terms. You mean by "marginal productivity theory" what I mean by the "limiting case".

I concede readily that "marginal productivity theory of wage" (in your language) has broken down and most be discarded. Using my language, however, we must regard the marginal productivity theory of demand for labor as a highly useful tool. For example, it helps us explain why, if (in the absence of unlimited competition among workers for jobs and through the exercise of union power or state power) the wage rate is set above the level which marginal productivity (marginal net revenue product) would have at full employment, the actual volume of employment will be smaller. Indeed, it is only the marginal productivity theory of demand that can help explain by how much the actual volume of employment will be reduced at the wage rate so fixed. The wage rate itself can be explained only by applying some additional principle, such as your power theory.

While, I believe, there is no disagreement between us on the essential points, we may ask whether my terminology is permissible. An analogy may help us in the answer. Would we ever say that the marginal utility theory breaks down and must be discarded when commodity prices are fixed by monopolists or by the government? Surely "marginal utility" will not in these cases explain the prices, but it will explain the quatities demanded at the prices fixed by monopoly or state power. What the marginal *utility* priniciple does in the theory of demand for consumers goods, the marginal *productivity* principle does in the theory of demand for productive factors. Thus I submit that there is a good justification for my terminology. Please forget or disregard what John Bates Clark wrote about marginal productivity, and do not blame modern theorists for what our predecessors may have "intended". The intention of marginal productivity theories in modern theory is *not* the explanation of factor prices.

III. REAL ISSUES

To make sure that we really agree on the real issues, I shall confirm my agreement on specific points:

(1) There may be, and usually is, wage rigidity even in the absence of labor monopoly.

(2) The monopoly power of unions cannot be explained entirely by their market position; a large part of the explanation will be in terms of sociological and political factors.

(3) In the course of technological progress, many changes in production technique will reduce, rather than increase, the marginal productivity of labor both at the actual employment level and the full empolyment level. This reduction may well be so drastic that even a large increase in the supply of capital may not suffice to offset it.

(4) Actual wage rates are often above the level which marginal productivity would have at the point of full employment; unemployment will arise as a result.

(5) To explain how wage rates can be raised to and maintained at such levels, we must resort to explanations outside "pure economic theory"; the power theory probably provides the principle for these explanations.

I may have missed some other real issues, but I have not noticed any point of essence on which I would disagree with you.

(*The Johns Hopkins University*)

DEPRECIATION ALLOWANCES AND GROWTH

KAICHI KIUCHI

I

Depreciation allowances are based ordinarily on the original cost of the assets concerned, and are limited cumulatively to the recovery of that cost. This arrangement is satisfactory in an era of relatively stable price levels, but can be seriously, and even ruinously, inadequate after a period of inflation. Under such conditions, the recovery in depreciated currency of the amount originally invested in a currency of greater purchasing power is only a fractional recovery in real terms. It does not suffice to offset real capital consumption.

There are, however, various levels at which those who are so inclined may care to meet this argument, as follows:

(1) They may raise the effect of technological advance and increases in productive efficiency.

(2) Some question may be posed as to the basic relevance of replacement requirements to the consideration of depreciation allowances.

(3) This view ignores the effect of physical growth on depreciation allowances.

The first level would cause no difficulty. It may be quite possible, in spite of inflated prices, to replace old assets with new ones whose dollar cost per unit of productive capacity or output is less than that of the cheaper but less efficient assets being replaced. On the next two levels, many accountants will insist that depreciation accounting is merely a device for allocating original cost and is entirely unrelated to replacement requirements. There is, however, the widely-held view that depreciation allowances necessarily fail to cover replacement costs during a period of rising prices. On the contrary, this view, it is argued, ignores the effects of physical asset expansion, by itself, would cause depreciation allowances to exceed replacement costs. It is possible, therefore, that the effect of rising prices on replacement cost may be

completely offset by the effect of growth on depreciation allowances.

This brief article will examine, from the above standpoint, the relationship between depreciation allowances and replacement costs through physical asset expansion or physical growth. Thus we shall note a few implications of depreciation allowances which have been hidden by the customary abstraction from the problem of growth.

II

On this subject, Robert Eisner, in his article entitled "Depreciation Allowances, Replacement Requirements and Growth" demonstrates by mathematical techniques that physical growth, by itself, would cause depreciation allowances to exceed replacement costs and concludes as follows;

A. Abstracting from price changes

(1) Depreciation allowances exceed replacement requirements in growing economics or growing firms.

(2) The excess of depreciation allowances over replacement requirements varies positively with (a) the rate of growth of gross investment and (b) the length of life of assets.

(3) The excess of depreciation allowances over replacement requirements varies negatively with the length of the period of amortization; accelerated amortization increases depreciation allowances.

B. When price changes are considered

(1) Increases in prices may not be sufficient to wipe out the excess of depreciation allowances over replacement requirements caused by growth in the real volume of investment.

(2) The extent of price increases necessary to cancel the effects of growth in the real volume of investment is a function of the rate of growth of investment (in monetary terms, which is a product of the real growth and the change in prices), the length of life of assets, and the period of amortization. Illustrative examples reveal that only when prices increase somewhat faster than real investment do replaeement requirements approach the magnitude of depreciation allowances.

Consequently, we may say that the phenomenon of growth places on shaky ground those who would argue that depreciation allowances are insufficient to meet replacement costs. In order to explain this analysis more clearly, let us add the following simplifying example being demonstrated by Felix Kaufman and Alan Gleason.[1]

Example

We shall deal with a single company operating over a period of eight years under the following conditions;

(1) The firm purchases one new depreciable unit each year.

(2) The price per unit of the depreciable asset remains constant at $1,000.

(3) The technology of the firm is constant, *i.e.*, replacements are physically the same as retirements.

(4) The period of amortization is two years and is identical with the length of life of the assets.

(5) The assets have no salvage value.

(6) The depreciation charge is established on a straight-line basis.

Exhibit I is designed solely to illustrate the effects of physical asset expansion on the relationship between depreciation charges and replacement costs. Consequently, an examination of Exhibit I shows that current depreciation either equals or exceeds the current replacement requirement in every year, with the excess occurring every other year. This means that working capital increases continuously by constant increments every other year.

Exhibit II is designed to introduce more realism, and at the same time to show the effects on working capital of increasing the percentage rate of change in investment. Exhibit II accomplishes this by scaling down additions in subsequent periods to a fraction of the initial investment and makes these annual increments a fixed per cent of the prior year investment. Except for this alteration no other changes are made in the assumptions of Exhibit I. The results in term of the effects on working capital are similar to the results of Exhibit I in that working capital shows a persistent accumulation due to the excess of current

depreciation charges over current replacement costs. In Exhibit II, however, the percentage rate of increase in working capital is greater than it is in the case of Exhibit I.

From Exhibits I and II the following conclusions can be drawn: (1) As long as a firm continues to expand, depreciation policy based upon acquisition cost will lead to the retention of current assets in excess of annual replacement needs and this excess amount will continue to glow.

(2) The greater the percentage rate of growth in fixed assets, the greater the percentage rate of growth in the amount by which current asset retention exceeds replacement needs.

Neither of the two preceding exhibits was intended to combines the effects of fixed asset growth with the effects of rising prices. This objective is accomplished in Exhibit III.

Exhibit I Equal Annual Increments of Investment Constant Prices
(Growth Increments Equal ¥1000 per Year)

Line	Description	Date	Years							
			1	2	3	4	5	6	7	8
1	Depreciable Assets on Hand	1/1	0	1000	2000	3000	4000	5000	6000	7000
2	Additions	1/1	1000	1000	1000	1000	1000	1000	1000	1000
3	Amount Subject to Depreciation	12/31	1000	2000	3000	4000	5000	6000	7000	8000
4	Assets Retired	12/31	0	1000	1000	2000	2000	3000	3000	4000
5	Assets Acquired for Replacement	12/31	0	1000	1000	2000	2000	3000	3000	4000
6	Balance of Asset Account (Line 3−4+5)	12/31	1000	2000	3000	4000	5000	6000	7000	8000
7	"Funds Provided" by Current Depreciation (Line 3×50%)		500	1000	1500	2000	2500	3000	3500	4000
8	"Funds Provided" by Prior Depreciation (Line 11–Prior Year)		0	500	500	1000	1000	1500	1500	2000
9	Total "Funds Provided"		500	1500	2000	3000	3500	4500	5000	6000
10	Funds used for Replacement (see Line 5)		0	1000	1000	2000	2000	3000	3000	4000
11	Cumulative Effect on Working Capital		500	500	1000	1000	1500	1500	2000	2000

* The Reader is cautioned to note that each year in the above table includes events at the beginning and at the end of the period.

Exhibit III is based upon the following important assumptions:

(1) Real growth increases at a constant annual rate of 10%. That is, each annual addition is 10% of the physical quantity of fixed assets existing in the preceeding year.

(2) Prices increase at a constant annual rate of 3%.

(3) The price index number at the top of each column in the table is used both in pricing additions acquired at the beginning of the year to which the column refers, and in pricing replacements purchased at the end of the previous year.

In spite of the introduction of the price increase factor, inspection of Exhibit III reveals that current assets retention exceeds the dollar cost of replacements. One should not infer from Exibit III, however, that price changes have no influence. The parameters were intentionally selected so that the growth effect on working capital retention would offset the price effect. Actually the price effect is stronger than the

Exhibit II Constant Percentage Increments of Investment Constant Prices
(Growth Increments Equal 10% per Year)

Line	Description	Date	Years							
			1	2	3	4	5	6	7	8
1	Depreciable Assets on Hand	1/1	0	1000	1100	1210	1331	1464	1610	1771
2	Additions	1/1	1000	100	110	121	133	146	161	177
3	Amonnt Subject to Depreciation	12/31	1000	1100	1210	1331	1464	1610	1771	1948
4	Assets Retired	12/31	0	1000	100	1110	221	1243	367	1404
5	Assets Acquired for Replacement	12/31	0	1000	100	1110	221	1243	367	1404
6	Balance of Asset Account (Line 3−4+5)	12/31	1000	1100	1210	1331	1464	1610	1771	1948
7	"Funds Provided" by Current Depreciation (Line 3×50%)		500	550	605	665	732	805	885	974
8	"Funds Provided" by Prior Depreciation (Line 11–Prior Period)		0	500	55	555	110	621	183	701
9	Total Funds Provided		500	1050	655	1220	842	1426	1068	1675
11	Funds Used for Replacement (Line 5)		0	1000	100	1100	221	1243	367	1404
10	Cumulative Effect on Working Capital		500	50	555	110	621	183	701	271

Exhibit III Constant Percentage Increments of Investment
Constant Percentage Price Increments
(Growth Rate=10%; Price Change =3%)

Line	Description	Date	Years								
			1	2	3	4	5	6	7	8	9*
	Price Level (at beginning of period)		100	103	106.09	109.27	112.55	115.93	119.41	122.99	126.68
	Unit;										
	Depreciable Assets on Hand	1/1	0	10	11	12.1	13.3	14.63	16.09	17.70	
	Additions	1/1	10	1	1.1	1.2	1.33	1.46	1.61	1.77	
	Retirements (−)	12/31	0	10	1	11.1	2.2	12.43	3.66	14.04	
	Replacements (+)	12/31	0	10	1	11.1	2.2	12.43	3.66	14.04	
		12/31	10	11	12.1	13.3	14.63	16.09	17.70	19.47	
	Dollars										
1	Depreciable Assets on Hand	1/1	0	1000.00	1163.90	1286.87	1489.69	1654.05	1908.58	2126.66	
2	Additions	1/1	1000.00	103.00	116.70	131.12	149.70	169.26	192.25	217.69	
3	Amt. Subject to Depreciation	12/31	1000.00	1103.00	1280.60	1417.99	1639.39	1823.31	2100.83	2344.35	
4	Assets Retired	12/31	0	1000.00	103.00	1177.60	240.39	1399.00	424.31	1676.52	
5	Assets Acquired for Replacement	12/31	0	1060.90	109.27	1249.30	255.05	1484.27	450.14	1778.58	
6	Bal. of Asset Acct. (Line 3−4+5)		1000.00	1163.90	1286.87	1489.69	1654.05	1908.58	2126.66	2446.41	
7	"Funds Provided" by Current Depreciation (Line 3×50%)		500.00	551.00	640.30	708.99	819.69	911.65	1050.41	1172.17	
8	"Funds Provided" by Prior Depreciation (Line 11-Prior Year)		0	500.00	(9.90)	521.13	(19.18)	545.46	(27.16)	573.11	
9	Total "Funds Provided"		500.00	1051.00	630.40	1230.12	800.51	1457.11	1023.25	1745.28	
10	Funds Used for Replacement (Line 5)		0	1060.00	109.27	1249.30	255.05	1484.27	450.14	1778.58	
11	Cumulation Effect on Working Capital		500.00	(9.90)	521.13	(19.18)	535.46	(27.16)	573.11	(33.30)	

* Ninth year price is shown since it is used to detemine the cost of replacements at the ends of the eighth year.

growth effect in the sense that if the percentage rate of increase in prices is the same as the percentage rate of increase in growth, working capital retention will be inadequate to meet replacement costs. In Exhibit III, a near-balance is achieved between the price and growth effects but only by postulating a growth rate which is about three times the rate of chahge in prices.

The principal conclusions of the above analysis may be summarized as follows:

(1) Physical asset expansion may offset the tendency of rising prices to cause replacement costs to exceed depreciation allowances.

(2) An expanding firm whose current depreciation allowances equal or exceed current replacement costs in a rising price period, has the option of taking action immediately to offset presumed future effects of the rising prices, or postponing action until current replacement costs exceed current depreciation allowances.

III

Rebert Eisner's view above mentioned endeavours to demonstrate a certain relationship between the rate of growth of acquisition of assets and the ratio of depreciation to replacement or retirement. This relationship reveals that the factor of growth tends to make depreciation allowances exceed replacement requirements and this excess tends to be more than enough to compensate for price increases, unless the rate of increase of prices is greater than the rate of growth of acquisition of real assets.

Before analysing the above-mentioned conclusions, let us summarize as simply and as briefly as possible the nature of depreciation and its place in income measurement. In this connection, it seems clear that the majority opinion among accountants is that the depreciation allowances are "to allocate cost of existing facilities, not to provide funds for replacements." In other words, depreciation is the cost of using capital assets and must be charged against the revenue obtained from their use if we are to get a reasonable measurement of net income. Thus depreciation allowances are based ordinarily on the original cost

of the assets concerned, and are limited cumulatively to the recovery of that cost. So that we may see the difference between this generally accepted and wholly logical concept of depreciation and the concept advocated by Robert Eisner as follows:

(1) Robert Eisner abandons the generally accepted concept of depreciation described in the preceeding paragraph—depreciation is not a matter of replacing physical assets after they wear out and are retired; it is a recognition of the usefulness of capital assets as they are literally consumed in operations.

(2) Robert Eisner confuses the replacement cost of assets now in use with the cost of replacement of assets formerly in use and retired during the current year—depreciation is the cost of using the assets now in use, not the cost of replacing assets used sometime in the past.

As mentioned above, Robert Eisner first assumed a contention that depreciation provisions should provide funds adequate for property replacement and then attempted to demonstrate that depreciation charges based on original cost will be more than sufficient to provide funds for assets replacement. There are, however, several questions involved here. Perhaps the most important of these is the fact that Robert Eisner assumes that the original outlay cost of the fully depreciated asset, which is identical with the full depreciation reserve, is the amount required to acquire the replacing assets. However, according to his own illustrations, he neither insists on closest correlation between original cost and replacement cost nor argues that the original cost of an asset as expressed in nominal dollars should determine the cost of replacement. His conclusion was that true replacement costs, as opposed to the price-adjusted accounting figures, are a very uncertain criterion for depreciation allowances, so that replacement costs are in fact merely conventional depreciation charges adjusted for price changes. Thus, we may say, Robert Eisner is merely on the conservative side defending conventional (original cost) depreciation against true replacement cost attackers.

Another approach to the problem is to make a comparison between the policy of depreciation on the basis of original cost adjusted for price

changes and the policy of depreciating on the basis of replacement value. In this case, they are all identical in assuming that the purpose of depreciation allowances is to provide funds adequate for property replacement, so that in the following two points:

(1) Investment in the business in continuous.

(2) Sufficient profits and its appropriate distribution.

However, the fundamental question to be considered here is the fact that accounting in replacement values assumes that investment outside the business is continuous and is in assets (including securities) which are subject to the same price rise as real assets in order to solve the financial problem posed. In so far as Robert Eisner is concerned, this point is neglected. So that in his conclusion, price changes have no influence. It is possible, however, for price change to overwhelm the impact of growth. Actually the price effect is stronger than the growth effect in the sense that if the percentage rate of increase in prices is the same as the percentage rate of increase in growth, working capital retention will be inadequate to meet replacement costs. Thus, we may say that Robert Eisner's conclusion is adequate only but when the rate of growth of acquisition of real assets is greater than the rate of increase of prices.

Next, as mentioned above, Robert Eisner tried to describe his idea directly from the phenomenon of growth. It seemed worth while to explore those relationships because American economy as a whole, and the majority of firms in American economy, have generally been growing. In a growing firm, if prices cover all costs, including depreciation, more funds will be made available to replace the expired services of assets consumed in operations in a given year than are required to replace the assets retired during the year. In all justice it must be, therefore, granted that had be restricted his discussion to the flow of funds through a firm, his argument might have been valid. Here is, however, an important thing to be considered. It is that Robert Eisner's theory of depreciation as the cost of replacing assets worn out by past use whose only relationship to the current year is that they happen to be replaced at this time, will not for a moment be accepted by anyone familiar with

the notion of depreciation as a cost of using the assets now in service. So that, as R. K. Mautz[2] indicated, it must be concluded that Robert Eisner has not demonstrated that conventional depreciation allowances are excessive or that profits are thereby misstated. Robert Eisner's failure to recognize depreciation as a measure of the expiration of the service life of assets now in use rather than the cost or replacing those assets used in the past, together with his confusion of the expression "replacement cost" of assets now in use with the "cost of replacing worn out assets when they require replacement" accounts for the fallacious conclusions to which he reasons. Thus it seems clear that not only does Robert Eisner advocate an inapplicable notion of depreciation, he also misunderstands quite completely the arguments of those who call for depreciation allowances based on the replacement cost of the assets now in service.

Finally, we are now in a position to examine the following assumptions Robert Eisner presented.

Simplifying Assumptions

(1) All capital assets have the same length of life.

(2) All capital assets have periods of amortization of the same length.

(3) Gross investment grows at a constant rate.

(4) All depreciation charges are made by the "straight-line" method, that is, by allocating an equal amount of the original cost of each asset to each year of its amortization period.

(5) Depreciation charges are begun for each asset at the beginning of the calender year following the year of its acquisition.

(6) There is no change in the general level of prices.

(7) All capital assets are of the "one-horse-shay" variety, giving the same level of service until their moment of extinction, at which point serviceability drops abruptly to zero.

With the aid of the assumptions presented above, Robert Eisner has concluded that depreciation allowances are too high and net profits, as well as net income and net investment, are understated by conventional accounting practices. Moreover, he has also suggested that economists

must avoid explaining our developing world with models which ignore the phenomenon of growth. However, the question to be considered here is that the assumptions presented above is in fact not realistic, so that Robert Eisner's conclusions are not always valid in theory or in practise.

IV

After this long investigation we arrive at the following conclusion.

Depreciation allowances are based ordinarily on the original cost of the assets concerned, and are limited cumulatively to the recovery of that cost. This recovery is satisfactory enough in periods of relative stability in the price level, but can be seriously, or even ruinously, inadequate during and after periods of inflation. Under such conditions, we cannot assume that "a dollar is a dollar." If it is, therefore, fluctuations in the value of money which call for adjustment, it is not relevant to take the price of the assets concerned, or to speculate, as many opponents of adjustment for varying prices have, on the impossibility if deciding when an asset will be replaced and what the cost will then be. Adjustment should be in terms of an index number of the general price level, and neither of indices of building costs and machinery prices, nor of a review of current prices of assets similar to those held.

As was already shown, Robert Eisner endeavours, from the above standpoint, to demonstrate the relationship between depreciation allowances and replacement costs through physical asset expansion or physical growth, and concludes that the factor of growth tends to make depreciation allowances exceed replacement cost and this excess tends to be more than enough to compensate for price increases, unless the rate of increase of prices is greater than the rate of growth of acquisition of real assets. It is possible, therefore, that the effect of rising prices on replacement cost may be completely offset by the effect of growth on depreciation allowances. It is natural, however, that a condition of extreme inflation would require modification of the above mentioned conclusion. Ultimately the central issue of this question is how much inflation is a prerequisite to action.

Next, as mentioned above, Robert Eisner tried to describe his idea must avoid explaining our developing world with models which ignore directly from the phenomenon of growth. In a growing firm, if prices cover all costs, including depreciation, more funds will be made available to replace the expired services of assets consumed in operations in a given year than are required to replace the assts retired during the years. It must be, therefore, granted that had be restricted his discussion to the flow of funds through a firm, his argument might have been valid. Here is how-ever, an important thing to be considered. It is that Robert Eisner's view of capital value erosion is in fact not valid. According to Robert Eisner, the object of depreciation policy is to provide funds adequate for property replacement, but the procedure he used is the same as well as in cost allocation theory of depreciation. It does not suffice, therefore, to offset real capital consumption. For it means that the method of allocating depreciation over the service life cannot properly be geared to the exhaustion of the services themselves, either physically or in terms of their value. Of course, this problem requires further discussion. The most important problem, however, is the distinction to be draw between the capital value of an assets and the sum of the realization values of its future services. Here is a proposition of cardinal importance for the understanding of depreciation.

References

1) F. Kaufman & A. Gleason, The Effect of Growth on the Adequacy or Depreciation Allowances, *The Accounting Review*, Oct., 1953.
2) R. K. Mautz, Another Look at Depreciation Allowances, *The Controller*, Jan., 1954.

LATENT UNEMPLOYMENT AND AGRICULTURE OF PEASANTRY

Chihiro Nakajima

I. INTRODUCTION

Japanese economy consists of two contrasting sectors, the one is capitalist-economic sector based on industry and the other noncapitalst-economic sector or home-economic sector that depends upon agriculture. There are many unbalances between these two but the rapid growth of Japanese economy owes ironically, a lot to these unbalances. One of them is the unbalance of labour income per head between the two sectors.

For example, during the period from 1934–1936 the net product per gainfully occupied in the industrial sector was ¥694.00 in average and the labour's relative share was 59%, hence the labour income per head was ¥410.00. Whereas the net product per gainfully occupied in agricultural sector was ¥170.00 with its relative share of 37%, hence the agricultural labour income per head was ¥63.00. However, there was an additional side-work income received from the nonagriculture which amounted to ¥27.00 per head. If this is taken into account, labour income per agriculture was about ¥90 per head but it is still only 22% against that of nonagriculture.

Assuming that the annual working days of nonagricultural labourer are about 300 days, while those of agricultural labourer about 150 days, then the daily labour income per gainfully occupied is calculated to be ¥1.37 for nonagriculture and ¥0.43 for agriculture.

Besides, farming labourers (peasants) had agricultural capitals, that are arable land and other reproducible capital, amounting to about ¥1800.00 and therefrom he earned the capital income amounting to ¥80.00. Consequently the total per-head income of agricultural labourer that he got as labour, capital and side-work incomes was ¥168.00 but it was only about 40% as against ¥410.00 of the employed labourer in nonagricultural industries.

This disparity of income between the two sectors has an inherent relation with another important problem, an existence of millions of latent unemployment in agriculture. Such basic picture of Japanese labor situation has persistently continued to be existing up to date throughout several decades of economic vicissitudes without undergoing any changes. Therefore these two problems must ultimately be analysed as a part of dynamic economy, but since it would be an extremely complicated task, a static analysis is made here and what is deemed as variables in the theory of dynamic economy such as population, capital, acreage of arable land, technique and so forth are assumed constant.

With this assumption the writer attempted here to make a static analysis or an interim analysis of these two problems in his belief that in light of the importance of these two problems even a static analysis on them would cast light for the study of Japanese economy.

The writer counts the following three reasons as causes for the disparity of labour income between said two sectors as well as the presence of latent unemployment in agriculture.

Namely,

1. In agriculture, the producing unit is "family farm" and so it is difficult to adjust employment at its own will.
2. Capital and arable land are insufficient as compared with population and industrial wage.[1]
3. Industrial wage is rigid downwards.[2]

II. ASSUMPTIONS ON THE DIFFERENCE BETWEEN AGRICULTURE AND INDUSTRY

In treating the captioned subject, I takes several assumptions.

A given country is assumed to consist of agricultural and industrial

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[1] Representing population as (a), industrial wage as (b) and capital and arable land as (c), (c) is too small as compared with (a) and (b), or (b) is too large against (a) and (c), or (a) is too large comparing with (b) and (c).

[2] The Japanese farming rent which is exceptionally higher than world level is forming a trinity together with the low standard of labour income and the presence of latent unemployment in agriculture sector and is an outcome of above stated causes. It is a mistake to take the high rent as a cause or a part of causes for the lower labour income in agriculture.

sectors. As to the differences between agriculture and industry, the following three assumptions are further made.

Assumption 1.

The factors of production in industry are the capital and the labour. The factors in agriculture are the farming land and the labour. In other words, the role of reproducible capital in agricnlture is to be neglected.

Assumption 2.

At a given technical stage in any country, the ratio of the combination of capital and labour in her industry is almost fixed. That is to say, the employment to be combined with unit capital in normal use is almost fixed, and the capital is always used at normal.[3] Therefore, the used amount of capital and the employment decreases proportionately in depression. On the other hand the ratio of combination between the farming land and the agricultural labour is variable. In other words, so far as the labour employment is concerned the capital in industry and the farming land is competitive or substitutive, while their combination ratio between the labour is fixed on the one hand and variable on the other. The farming land is used to the maximum extent irrespective of whether in prosperity or depression.

Assumption 3.

The productive unit in industry is "firm" and the same in agricultnre is "family farm" — a home-economy. Upon this difference the following two assumptions are made.

Assumption 3, 1.

There is distinction and confrontation between employer and employee in firm. Firm employs only minimum labour that is required. It dismisses some of its labourers and decreases employment in depression, whereas there cannot be such distinction and confrontation in family farm. Therefore, when a member of a farming family cannot

[3] There would be an actual case where a unit capital is combined with the labour beyond at normal ratio, but it does not upset the logics of this treates even such cases is included.

find any employment in industry, the family takes him in with the spirit of mutual help based on the family system even though he is an excessive labour that is not required. Namely, the industrial sector employs as it requires, adjusting employment *at its own will* and pushes any excessive working population against agriculture. (here, for sake of convenience, another assumption is made that labourers employed in industry come from family farms and they can go back to them.) The agricultural sector takes in the labourers pushed back to it. Thus the employment in agriculture is *dependently* decided and changed.

Now, if the working population of the conutry is represented by $\bar{N}$ ($\bar{N}$ is const.), and the employment in agriculture and industry respectively by N_1, N_2,

$$N_1 = g(N_2) = \bar{N} - N_2$$

hence, when N_2, emyloyment in industry decreases during depression N_1, employment in agriculture increases just as much.

Assumption 3, 2.

Firm reduces employment as well as production during depression, in its efforts to maintain the product price, whereas the family farm cannot reduce employment as well as output even during depression, on the contrary it rather increases them, and consequently the price of agricultural products can not help but fall. Therefore, the adjustment between effective demand and supply in industry is made by controlling the output until the upper limit of the productive capacity is reached. While, the adjustment in agriculture is made by the fluctuation of price.

III. DEFINITIONS OF LATENT UNEMPLOYMENT

According to the aforesaid assumption 3, 1, all workers are to be either employed by firm or absorbed into farm. Here, another assumption is set that the workers thus taken in by farms engage exclusively in farming. Then, from these two assumption it is inferred that no visible unemployment exists in the both sectors. Two assumptions are further added ; one is that all the workers are ($\bar{N}$) homogenous and the other that those who are employed by industry (N_2) get same wage (W).

Since the latent unemployment, as stated hereafter, is defined with industrial wages as its basis, there cannot be any latent unemployment in industry in so far as all workers are assumed to be paid same wage. Accordingly, in the model of this treatise, if the latent unemployment should exist at all, it must be only in the field of agriculture.

If the agricultural output is indicated as O_1, under the assumption 1, O_1 is the function of its employment N_1 as well as of the total farming area. However, as the total farming area is constant, O_1 becomes the function of N_1 only. This relation is expressed as $O_1=f(N_1)$. If the price of agricultural product is represented by P_1, then $P_1f'(N_1)$ means the marginal productivity of farming labor and also the agricultural labor income per head.[4]

Next, we call such agricultural employment that makes labor incomes in both agriculture and industry equal, in this sense, "normal employment in agriculture" and indicate as N_1^*. Then, $P_1f'(N_1^*)$ equals W. The larger is the farming area of the country, the bigger would be such N_1^*. And N_1^* would increase as P_1 rises, decrease as W rises.

Now let us denote the actual employment of agriculture by N_1 against the said normal employment N_1^*. Then, in case where $N_1>N_1^*$, $P_1f'(N_1)<W$, that is, agricultural labor income gets smaller than industrial wage. Moreover, the difference between the actual employment and the normal employmet in agriculture, i.e. $N_1-N_1^*=Z$, is the quantitative definition of latent unemployment emerging in agriculture. What, then, is the qualitative definition of that ?

In the model of this treatise, all agricultural family labourers get same labor income expressed as $P_1f'(N_1)$. If, therefore, $N_1>N_1^*$, viz.

[4] P_1f' is the marginal productivity of agricultural labor and does not immediately mean the agricultural labor income. The reason is that the marginal productivity of labor in equilibrium, so far as firms are concerned, equals wage (labor income), whereas the labor income in a farming house is not something to be paid as a price but the residual after having paid the rent. Such residual is not necessarily equal to the product of the quantity of family labor and its marginal productivity. The residual can be equal to the said product only when the following particular condition is satisfied, viz. the condition that the production function of family farm, in which the output is a function of area of land and labor, is a homogeneous function of 1st degree.

$P_1 f'(N_1) < W$ is the case, *all* members of N_1 obtain less labor income than W. Accordingly, if latent unemployment is defined as such workers that receive less labor income than W_1, the total farm workers must become latent unemployment, which is contradictory to the quantitative definition above referred to. Now, the qualitative definition of latent unemployment that is yet compatible with its quantitative definition is as follows. This is, latent unemployment is "the excess labourers in agriculture" in that sense that the removal of such number of labourers from agriculture makes the labor incomes in both sectors equal. Such latent unemployment, $Z = N_1 - N_1^*$, would be larger as N_1 becomes larger, as the farming area of the country is smaller and as the price of agricultural products (P_1) is lower.

IV. EMERGENCE OF LATENT UNEMPLOYMENT

The capital amount of a country at a given time is definite. And since the ratio of combination between capital and labor power is assumed to be constant, the employment that can be combined with the total capital is also constant. If we denote this employment as $\bar{N}_2$, it means the upper limit of employment in the industrial sector. Therefore, where there is enough effective demand for industrial products, the capital is used to its maximum extent and simultaneously industrial employment reaches $\bar{N}_2$. However, where the effective demand for the product is short, the employment in industry is less than $\bar{N}_2$. This means that the actual employment in industry, that is N_2, is determined by the effective demand for its products. Hence $N_2 \leqq \bar{N}_2$. And out of the total workers $\bar{N}$, only N_2 is employed by industry and the rest, $\bar{N} - N_2$, is brought to farm house and works there. Therefore, the actual employment in agriculture N_1 is equal to $\bar{N} - N_2$. Then in the case of $N_1 > N_1^*$, there appears the latent unemployment which is as large as

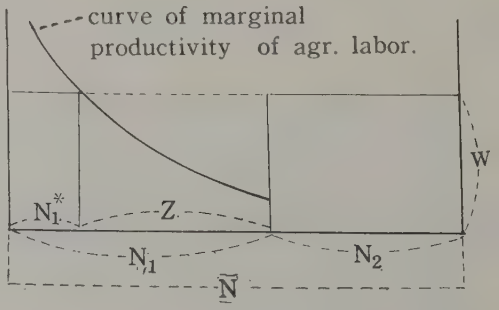


Fig. 1

$Z = N_1 - N_1^*$ under the aforesaid definition and, further, $Z = N_1 - N_1^* = \bar{N} - N_2 - N_1^*$. This relation is illustrated in Fig. 1.

Then, if $N_1 \leqq N_1^*$, what will happen there ? First, where $N_1 = N_1^*$, there is, of course, no latent unemployment. Second, where $N_1 < N_1^*$, the agricultural labor income becomes higher than industral wage and it would naturally cause migration of laborers from industry to agriculture. Therefore, in order that industrial sector secures employment as much as N_2 or nearly it, it is necessary for industry to raise its wage up to the level of agricultural labor income.[5] And when the labor incomes in the both fields come to be equal as the result of such wage increase in industry, no latent employment will then exist.

Fig. 2 shows how this latent employment changes in prosperity. In industry, the employment will reach its upper limit $\bar{N}_2$ at the same money wage as before in accordance with increase of effective demand for its products, while employment in agriculture decreases by as much as it increased in industry. Accordingly, if labor hours for agricultural workers are assumed to be constant, agricultural output cannot help being reduced because number to be combined with same area of farming land is decreased. On the other hand, demand for argricultural product increases, hence its price P_1 rises. However, it is considered that a certain upper limit of such price rise of agricultural product is set through international trade. If this limit is expressed as $\bar{P}_1$, the marginal value-productivity of agricultural labor moves up from $P_1 f'$ up to $\bar{P}_1 f'$, namely the curve of marginal value-productivity is shifted rightwards. Accordingly, N_1^*, or such normal employment in agriculture that makes the marginal value-productivity of it equal to industrial wage W, increases itself. If the normal employment

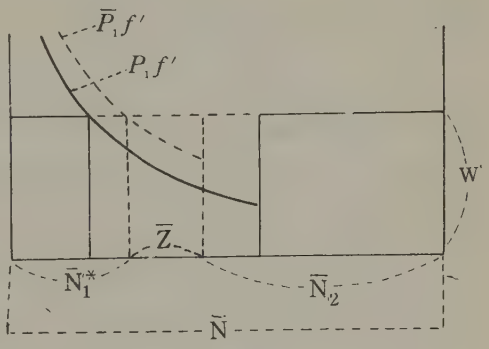


Fig. 2

[5] It would be advisable to assume that the money wage W is inflexible downwards and flexible upwards.

in agriculture at the price $\bar{P}_1$ is expressed as $\bar{N}_1^*$, such $\bar{N}_1^*$ is the upper limit of N_1^*. Therefore, in the case of $\bar{N} > \bar{N}_2 + \bar{N}_1^*$, the latent unemployment as large as $\bar{N} - \bar{N}_2 - \bar{N}_1^*$ remains even in the time of business prosperity. Thus, in the expression $\bar{Z} = \bar{N} - \bar{N}_2 - \bar{N}_1^*$, $\bar{Z}$ is the lower limit of latent unemployment Z. If this $\bar{Z}$ is zero or negative, it means that latent unemployment will disappear in good times.

V. TWO TYPES OF LATENT UNEMPLOYMENT

In such a country that where latent unemployment does exist even in good times, the latent unemployment in the depression is supposed to consist of two types. One is the unemployment which disappears with the return of prosperity, while the other is the one which continues to exist even during good times.

If the total quantly of latent unemployment in the depression is denoted by Z, and the employment of industry in the depression by N_2 and normal employment of agriculture at the agricultural price in depression by N_1^*, then

$$Z = \bar{N} - (N_2 + N_1^*) = \{\bar{N} - (\bar{N}_2 + \bar{N}_1^*)\} + \{(\bar{N}_2 + \bar{N}_1^*) - (N_2 + N_1^*)\} .$$

Whereas, $\bar{N} - (\bar{N}_2 + \bar{N}_1^*) = \bar{Z}$.

Thus, by substituting $\bar{Z}$ for $\{\bar{N} - (\bar{N}_2 + \bar{N}_1^*)\}$ in the above expression we get

$$Z = \bar{Z} + \{(\bar{N}_2 + \bar{N}_1^*) - (N_2 + N_1^*)\} .$$

The second term of the right side of the expression i.e., $\{(\bar{N}_2 + \bar{N}_1^*) - (N_2 + N_1^*)\}$ indicates latent unemployment due to depression, or "cyclical latent unemployment," which can be removed by the increase of effective demand.

While the first term of the right side of that, $\bar{Z}$, indicates the latent unemployment which does not disappear even by the increase of effective demand, and in the sense it should be called "structural latent unemployment" or latent unemployment due to scarcity of capital and arable land, in other words, scarcity of complementary productive resources for labor force.

The reason why it is called as such is that if total quantity of

capital is greater, the larger will be $\bar{N}_2$, namely the maximum employment of the industry, likewise, if acreage of arable land is greater, the marginal productivity curve of the agricultural labor will be shifted to the right, and thus, in Fig. 2, the part $\bar{Z}$ in between $\bar{N}_2$ and $\bar{N}_1{}^*$ will disappear.

Therefore, setting aside the effective demand, when abundant supply of industrial capital is available there will occur no shortage of arable land, even if arable land is actually limited. Likewise, when arable land is abundantly available, there will be no shortage of capital, even if the available capital is actually small.

As mentioned above, in regard to the employment of labor population, capital and arable land are in a competitive or alternative relation, therefore, the shortage of the former means the shortage of the latter in itself. The latent unemployment due to the shortage of capital and arable land could be understood as a relative excess in population as against capital and arable land.

According to the exdression of Mrs. Robinson, the part $\bar{Z}$ of latent unemployment is that of Marxian type and the rest is that of Keynsian type. As far as our country is concerned the latent unemployment of marxian type has always been much important.

VI. CONDITIONS OF EMERGENCE OF STRUCTURAL LATENT UNEMPLOYMENT

There is few noteworthy problems as to the conditions of emergence of latent unemployment caused by depression. Thus our study shall be directed toward the conditions of latent unemployment caused by shortage of capital and arable land.

Whether or not capital supply and arable land is sufficient for the total labor force cannot be determined by the relations between the relative sizes of these three elements alone, but is determined by the relation between these three and the specific level of income, i. e., according to the conception herein employed, the industrial wage level expressed by "W".

If the supply of capital and arable land is deemed to be insufficient against the population of labor, it should be quite in the sense that the

capital and arable land is insufficient to let all the members of labor population earn the wage as high as *W*.

Therefore, when the size of labor population, supply of capital and arable land are kept constant, it is possible that the capital supply and arable land are in short against working population at a certain wage level while at a lower wage level they are sufficient. Therefore, in the former case latent unemployment comes into existence while in the latter case no latent unemployment emerges.

In other words, the fact that there exists latent unemployment in the agriculture when industrial wage rate is at a given level means that there is so much of excess supply of labor and that nevertheless the industrial wage rate does not go down. If the wage rate is as the prices of general commedities, when there is excess-supply of labor, the wage rate will continue to go down to the point at which the demand and supply of labor just meet with each other and so latent unemployment will disappear.

However, the wage rate is inflexible, and consequently excess-supply of labor exists chronically. Therefore, one of the factors supporting the existence of latent unemployment consists in the fact that industrial wage rate is inflexible. On the other hand, if each productive unit of the agriculture were a firm it would not hire any more labor than it actually needs, and thus there will not be latent unemployment in agriculture but *revealed* unemployment comes into being. Thus, one of the factors which makes latent unemployment possible lies in the fact that each productive unit of the agriculture takes in more labor than they actually need.

Thus the conditions of emergence of structural latent unemployment could be summarized as follows:

1. The productive unit of the agriculture is each farming household, and therefore, the adjustment of employment in agriculture is impossible, if it was left to be alone.
2. In comparison with labor population and industrial wage level, capital and arable land is too small. In other words, industrial wage rate is too high in comparison with labor population as well as the supply of capital and arable land.

3. Industrial wage rate is inflexible toward down.

These three factors are the causes for latent unemployment and low labor income as well as for high land rent of agriculture. It would then be questioned that why industrial wage level is fixed at such a high level at which all labor population cannot be employed and why it does not go down.

This question can be answered only by the "Social Power Theory" established by Dr. Yasuma Takata.

VII. POLICY TO ELIMINATE LATENT UNEMPLOYMENT

Latent unemployment due to depression can be eliminated by Kenzian policy which stresses the increase of effective demand.

However in such case as in our country, it is obviously impossible to completely remove latent unemployment through such policy.

To cope with structural unemployment, the following countermeasures could be thoughl of, althought there will be considerable difficulties to practise these measures, and they will have no immediate effect upon the improvement of the situation.

The first measure may be the restraint of increase of population, birth control and immigration, etc.

The second measure may be the promotion of capital accumulation in industry. Thirdly, as for agriculture, the following three measures are thought of:

(A) Extensive and intensive enlargement of arable land, in other words, investment to improve the land. (B) Accumulation of agricultural capital complementary for labor force, for instance, to increase milk cow and other live-stock, fruit trees and equipment to process agricultural products[6]. (C) Devices and its practise of agricultural technics and management systems which moderate the laws of dimishing return of land. Here, (C) and (B) are interrelated with each other.

[6] The increase of agricultural capital which substitutes labor force., for instance agricultural machines are effective to raise welfare of farming family, but will have no power to elevate marginal productivity of farmiug family, but will have no power to elevate marginal productivity of farming labor at a given level of employment and thereby decrease latent unemployment.

Agriculture is specifically a seasonal industry precarious to the act of nature, and therefore in slack season the marginal productivity curve of labor shifts to left and latent unemployment increases. Also to cope with such special circumstances, devices and improvement of management system are wanted and they are effective to the purpose.

ON THE DETERMINATION OF THE OPTIMAL OPERATING LEVEL IN A TEXTILE MANUFACTURING FACTORY

YUTAKA OSAWA

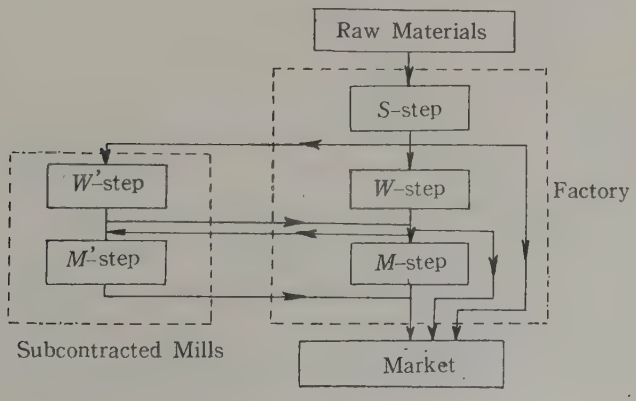
SUMMARY

This paper deals with the derivation of optimal operating level of a factory in which many brands of textile goods are manufactured. Linear programming technique is applied to resolving the conflicting requirements among market prices, manufacturing costs and maintenance expenses of the factory, with which the firm is faced. In setting up the processes, we encounter with the decreasing cost problem. We proceed by separating the entire program to an equivalent set of programs. Formulation of the model shows how the change of condition caused by reduction of production levels can be studied as linear programs.

THE PROBLEM

Outlook of the production mechanism in this factory is presented in Figure I. First, raw materials are put into the "*S*–step" preparing the first type of products, yarn. A portion of the products may be supplied to markets (either domestic or overseas) and the remainder is transferred to the next "*W*–step", in order to make the second type of

Figure 1.



products, fabric. In this step, the firm can make use of machines in subcontracted mills instead of their own. "W'–step" shows the use of these facilities. The fabric is either sold in market in that form, or given the finishing touch in the last "M–step", where the firm may also give orders to subcontracted mills, which are shown by "M'–step". This final step mainly consists of dyeing works.

These subcontracted mills are so–called "medium and small enterprises", occupying a greater part of the industrial firms in Japan. Generally speaking, they have a relatively fixed connection with this particular firm, and their manufacturing cost is usually moderate relative to its own, even though the charges of packing, transportation, etc. of materials are included. It can be said in some sense that they play a role as a safety valve for their powerful parent company.

Raw material, that is raw cotton, is divided into three kinds of mixture, A, B and C, and a part of the B receives a special treatment in the S–step to give B'. Every commodity is produced by making use of any one of these kinds of mixture, and is designated by a letter showing its final manufacturing step with a suffix, say $S_1, \cdots, S_4$; $W_1, \cdots, W_3$; $M_1, \cdots, M_{14}$.

The purpose of the programming is to decide the production level of these 21 commodities which maximizes the profit gained, and this decides simultaneously the choice between this concerned factory and its subcontracted mills with regard to a step, or in other words, the operating level of this factory. In this paper, the term, operating level, is used independently for every step, not as a whole, since it can be regarded as a separate function in either operation and organization. More concretely, for the W–step, it is represented as a percentage of the total number of machines when it is operated in normal condition during the regular working hours (8 hours per day, 25 days per month), and for the M–step, a percentage of the possible amount in yards manufactured during the regular working hours. The S–step is excluded from this programming for a convenience, for it is free from regulations except specifying the amount of B'.

Now we can see the situation of the textile industry when the

programming was made as follows. Market condition of the specific fibre manufactured by this factory was very poor. Reduction of demand in domestic and overseas markets caused decline of their market prices, and most of the brands got into the red figures. "Yarn is profitable, fabric is losing", a general tendency in this type of industry, came under here, and most commodities except S_i were produced in bleeding.

Therefore, from this point of view, it is preferable for the firm to sell more yarn, that is to increase the production of S_i. Moreover, as mentioned above, the manufacturing cost in subcontracted mills applicable to the production of W_i and M_i, is relatively cheap. For these reasons, it turns out to increase the loss of the firm to work the machines in the W– and M–steps.

However, when we reduce the operating level in both steps, it necessarily makes the cost dear with respect to other points. It is very difficult to discharge the surplus workers arising from the reduction of operating level under the present circumstances of labor contract in Japan. Now, if we could regard the wages as being fixed, the problem would be rather simple. But in these cases, the firm takes such means as suspension of filling up the spontaneous retirement, transposition of personnel to another step or another factory within the firm. The latter means will not be taken so far as the change of the operating level is relatively small, but when it amounts to a certain extent this will be taken. Even in this case, it can not be expected that the reduction of workers is proportional to that of operating level, and the number of workers per machine tends to be larger as the level reduces. Thus we may say that the wage, as a whole, is neither fixed cost, nor proportional cost. This is only one example. There are many other cost items having such properties. These situations seem to be common in most textile industry in Japan.

Under such circumstances, the profit per unit of product W_i and M_i produced by this factory is very low or the loss is extremely high. Let us exclude the operating level under 60 per cent from our consideration, since it means cessation of the entire operation. Then it is the fundamental question for this programming to see the extent of the

operating level in this factory to give the maximum profit under these circumstances.

It is important to note that the factor of time element required for production is ignored here. In practice, it takes about a month for S_i, two for W_i, and three for M_i, to finish the work. In the textile industry, the price fluctuation is particularly violent as everybody knows. Therefore, to be more realistic, the dynamic formulation should be applied taking into account these manufacturing period. In our formulation, however, to avoid a complexity, the problem is, as a first approximation, dealt with for only one section in the flow of production, by means of adopting forecasted selling prices at the future time when these products will be carried on transactions actually.

FORMULATION OF MATHEMATICAL MODEL

Prior to the construction of processes, let us assume that all the machines in the *W*– and *M*–steps of this factory can be used under the same condition within some ranges of the operating level of each step. Here the term "machine" means not only the machine itself but also all accessories and workers belonging to it inclusive.

Concerning this assumption, two points should be noticed. The first is a technological one. There are several kinds of machines different in widths, types and efficiency in the factory, but from the technological point of view it can be said that a possible inefficient use of mahhines due to this assumption brings about negligible changes in the solution. In the above assumption, we say that all the machines can be used *under the same condition*, but there is one exception. For the *W*–step, we have a difference in the number of machines required for producing a unit of any fabric. This figure is shown below by a given coefficient, but we regard it as being constant for any operating level of the *W*–step, or for any machine in this step. For the *M*–step and the steps using the subcontracted mills, we do not allow even for this kind of difference, but we assume that all the machines are applicable for all products under the entirely same condition.

The second is more important and rather theoretical. Primarily, a process means a given combination of inputs and outputs relating to an

activity. We should take all their components into consideration, and different proportion of components makes a different process. In this case, we might make distinguish among machines, their accessories, the workers, the motive power, etc., but for the sake of manageability of the problem, we have to make a compromise to some extent. Thus we make them into a bandle and represent them as one factor, "machine". However, this introduces some difficulties in another point. As pointed out in the last section, the number of workers per machine increases when the firm reduces the operating levels. This implies that the factors involved in the term "machine" do not change proportionally. Any machine is equally applicable to any product at the same technological condition, but depending on the operating level of the step we have a machine different, as to speak, in quality, or in cost. The phrase "within some ranges of the operating level" in the above assumption is connected with this point. We assume these ranges are shown by every 10 per cent cut of the operating levels in each step. (We exclude the operating levels under 60 per cent as mentioned above.) This figure is based mainly on the policy of the firm concerning the transposition of personnel, the number of workers being a major non-proportional factor. Thus we have a constant combination of factors within these ranges, and this, in turn, brings us separate processes which are different in profit brought about by their activity depending on these ranges of the operating levels.

Now with these assumptions, we can construct 39 processes as shown in Table I. The entries in this table are valid for any operating level of each step. The column headed by "steps applicable" lists the steps which we specify in advance as the object of our actual choice in the programming. Usually, we can set up two processes, W and W' for every W_i, and four processes, W–M, W'–M, W–M' and W'–M' for every M_i, but for some products we can not make use of the steps in subcontracted mills since the special techniques are required in their production, and for some products we are enforced to make use of subcontracted mills for financial or political reasons. So we specify the feasible steps in advance without including all their possibilities.

Table I. Construction of Processes.

Final step	Process number	Name of product	Market *	Mixture	Amount of raw materials (Pounds per 1000 yards)	Steps applicable		Number of machines required to produce 1000 yards fabric in the W-step
						W	M	
W	1	W_1	O	A	250	W'	—	—
	2	W_2	O	A	210	W	—	0.625
	3		O	A	210	W'	—	—
	4	W_3	D	B	180	W	—	0.667
	5		D	B	180	W'	—	—
M	6	M_1	O	A	300	W'	M'	—
	7	M_2	O	A	315	W	M	0.850
	8		O	A	315	W	M'	0.850
	9	M_3	O	A	280	W'	M	—
	10		O	A	280	W'	M'	—
	11	M_4	O	A	250	W	M	0.656
	12		O	A	250	W	M'	0.656
	13		O	A	250	W'	M	—
	14		O	A	250	W'	M'	—
	15	M_5	O	A	250	W	M'	0.656
	16		O	A	250	W'	M'	—
	17	M_6	O	A	250	W	M	0.680
	18	M_7	D	B	180	W	M	0.672
	19		D	B	180	W	M'	0.672
	20		D	B	180	W'	M	—
	21		D	B	180	W'	M'	—
	22	M_8	D	B	360	W	M'	0.811
	23		D	B	360	W'	M'	—
	24	M_9	D	B'	360	W	M'	0.842
	25		D	B'	360	W'	M'	—
	26	M_{10}	D	B	480	W'	M	—
	27		D	B	480	W'	M'	—
	28	M_{11}	D	B'	460	W'	M'	—
	29	M_{12}	D	B	400	W'	M'	—
	30	M_{13}	D	B'	400	W'	M'	
	31	M_{14}	D	B'	480	W'	M'	—
S	32	S_1	O	A	—	—	—	—
	33		D	A	—	—	—	—
	34	S_2	O	B	—	—	—	—
	35		D	B	—	—	—	—
	36	S_3	O	B'	—	—	—	—
	37		D	B'	—	—	—	—
	38	S_4	O	C	—	—	—	—
	39		D	C	—	—	—	—

* O denotes a product for overseas markets, D for domestic markets.

Now these 39 processes are not sufficient to formulate the entire problem. For, in any process making use of the *W*– and *M*–steps we have to make distinguish among them depending on the operating level of the step even though their technological coefficients are the same. Theoretically, we might formulate the processes using the first 60 per cent machines, the processes using the additional 10 per cent machines, ···, separately for any product relating to either the *W*– and *M*–step.

However, this brings us so many processes that we are hardly manageable them. Instead, we solve the problem by dividing the entire system into an equivalent set of programming. Let us consider that the firm is operating at, say, 90–100 per cent level for both the *W*– and *M*–steps. Then in these ranges we have the 39 processes as shown in Table I, of which costs are proportional to their level of activity. Thus we have one linear programming problem in a usual sense. Such problems can be formulated for any combination of the operating level in the *W*– and *M*–steps, and we have 16 separate programming problems in all. In any separate problem we have a same technological matrix, but when we move from one to another, we have only the differences in profit functions and operating level limits of the *W*– and *M*–steps. Such a solution of programming as to give the maximum value of the maximized profits is apparently equivalent to the solution of the whole system aimed at.

Through such a device we now proceed to formulate a linear inequality system. We have a series of constrained conditions which are common to all separate programs except (4) and (5) below.

(1) The total amount of raw cotton put into this program is to be equal to 1,295,491 pounds. This equality is due to the fact that this corresponds to a part of the whole program of the firm and that purchasing raw cotton is limited by the export linkage.

(2) Mixtures *A* and *B* can not be used over 741,000 and 508,720 pounds, respectively. The *C* is to be equal to 70,000 pounds.

(3) The amount applicable for the *B'* in the *B* is to be less than 48,290 pounds.

(4) Machines applicable for the *W*–step are 614 in number. For

Table II Linear Equality System

Limitations \ Vector		Structural Vector																													
		P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}	P_{17}	P_{18}	P_{19}	P_{20}	P_{21}	P_{22}	P_{23}	P_{24}	P_{25}	P_{26}	P_{27}	P_{28}	P_{29}	P_{30}	P_{31}
Resource Limitations	(1)	250	210	210	180	180	300	315	315	280	280	250	250	250	250	250	250	250	180	180	180	180	360	360	360	360	480	480	460	400	400	480
	(2) A	250	210	210			300	315	315	280	280	250	250	250	250	250	250	250														
	B				180	180													180	180	180	180	360	360	360	360	480	480	460	400	400	480
	C																															
	(3) B'																								360	360			460		400	480
	(4)		.625		.667			.850	.850			.656	.656			.656		.680	.672	.672			.811		.842							
	(5)							1		1		1		1				1	1		1						1					
Market Limitations	(6)																								1	1			1		1	1
	(7)	250	210	210	180	180	300	315	315	280	280	250	250	250	250	250	250	250														
	(8) W_1	1																														
	W_2		1	1																												
	W_3				1	1																										
	M_1						1																									
	M_2							1	1																							
	M_3									1	1																					
	M_4											1	1	1	1																	
	M_5															1	1															
	M_6																	1														
	M_7																		1	1	1	1										
	M_8																						1	1								
	M_9																								1	1						
	M_{10}																										1	1				
	M_{11}																												1			
	M_{12}																													1		
	M_{13}																														1	
	M_{14}																															1

Continued

								Slack Vector																										Requirement Vector
P_{32}	P_{33}	P_{34}	P_{35}	P_{36}	P_{37}	P_{38}	P_{39}	P_{40}	P_{41}	P_{42}	P_{43}	P_{44}	P_{45}	P_{46}	P_{47}	P_{48}	P_{49}	P_{50}	P_{51}	P_{52}	P_{53}	P_{54}	P_{55}	P_{56}	P_{57}	P_{58}	P_{59}	P_{60}	P_{61}	P_{62}	P_{63}	P_{64}	P_{65}	P_0
1	1	1	1	1	1	1	1																											1,295,491.0
1	1							1																										741,000.0
		1	1	1	1				1																									508,720.0
						1	1																											70,000.0
				1	1					1																								48,290.0
											1																							614.0
											1	1																						61.0
													1																					900.0
													1	1																				90.0
															1																			90.0
1		1		1		1										−1																		226,971.8
																	1																	50.0
																		1																240.0
																			1															100.0
																				1														50.0
																					1													60.0
																						1												100.0
																							1											860.0
																								1										350.0
																									1									50.0
																										1								100.0
																											1							48.0
																												1						20.0
																													1					30.0
																														1				20.0
																															1			100.0
																																1		48.0
																																	1	20.0

any separate program, 10 per cent cut of this forms the upper and lower limits of the adoptable range of machines.

(5) We have the upper and lower limits for the M–step similarly. In this step the firm can give the finishing touch to the fabric up to 900,000 yards under the normal conditions. So the limits are given by every 10 per cent cut of this amount for any separate program.

The above five conditions are the "technological" or so-called resource limitations of the program. In addition to them, we have the following "market limitations":

(6) Of the products M_i, the products using the mixture B' all of which are sold in the domestic markets, has a limited demand. This amounts to 90,000 yards according to the forecast by the officer of the firm.

(7) Products for overseas markets are to be more than 20 per cent on the basis of pounds. This limitation is partly due to the export linkage and partly aims at maintaining the overseas markets.

(8) All products except S_i have the upper bounds by which the firm is able to sell them in markets.

The above limitations are expressed as shown in Table II. Let us designate the processes given in Table I by P_i, $i=1, \cdots, 39$, the slack activity by P_j, $j=40, \cdots, 65$, and the requirement vector by P_0. Then the Table II is to express the linear equality system $\sum \lambda_i P_i = P_0$, where λ_i represents the level of activity in each process. Here the limitations for (4) and (5) show the case of 90–100 per cent operating level of the W– and M–steps, respectively, and the unit of variable λ_i is 1000 yards far $i=1, \cdots, 31$ and pounds for $i=32, \cdots, 39$.

Finally, we have 16 separate objective or profit functionals for each separate program, as mentioned above. They are given by $\sum c_i \lambda_i$, where c_i is the unit profit of the i–th process with the exception of the genuine fixed costs. The c_i for each separate program is determined by the following form:

$$c_i = p_i - (q_i + \alpha w_i + \beta m_i),$$

where p_i is the selling price of the product relating to the i–th process, q_i is the total cost of the i–th process excluding only the manufacturing

costs in the W– and M–steps, w_i and m_i are the manufacturing costs in the W– and M–steps respectively when they are operated at the full level (i. e. each 90–100 per cent level), and α, β are the values determined corresponding to their level, respectively. Thus for the products S_i, w_i and m_i are always equal to zero, for the W_i, m_i are equal to zero. The values of α and β are given in Table III. These values are determined taking into consideration the fact that the present poor market condition will continue over a considerable length of time. The c_i's calculated in such a way for the separate program shown in Table II are given in Table IV. They will more or less throw light upon the relative role of products.

Table III.

Operating level (%)	α	β
90–100	1	1
80–90	1.08	1.05
70–80	1.18	1.11
60–70	1.30	1.20

Table IV.

Process number	0	1	2	3	4	5	6	7	8	9
0	—	−835	−1435	−1575	−2725	−2450	−6245	−5465	−4070	3325
10	3605	−5595	−3085	−4955	−2445	−3030	−2470	−1445	−7510	−4720
20	−6330	−3540	2145	3030	4810	5695	385	795	3700	−320
30	285	−12115	0	1.5	0	1.5	0	0	7.5	7.5

SOME NOTES ON THE SOLUTION

From the above mentioned formulation, we can solve the entire program as a set of problems of maxizing $\sum c_i\lambda_i$ subject to $\sum \lambda_i P_i = P_0$, $\lambda_i \geqq 0$, $i=1, \cdots, 65$). For each of 16 programs, we have the profit maximized as shown by the following table. Concerning the programs having no entries in the table, we can see at once by checking the structure of processes that they give no more profit than that obtainable at 90–80 per cent and 70–60 per cent operating levels in the W– and M–steps respectively. Thus we have found that the optimal solution is given at these levels.[1] The details of the optimal solution are shown in Table VI. There are three solutions giving the same maximum profit. Therefore any linear combination of them is also the optimal solution.

[1] Perhaps a device will be worked out for solving the problem more efficiently through the similar procedure as the method of "extended simplex tableau" developed by Charnes and Lemke.

Table V. Maximum profits for separate programs, (¥ 1000)

Operating levels (%)		*M*-step 90–100	70–80	60–70
W-step	90–100	−1091	—	−710
	80–90	—	—	−664
	70–80	—	−974	−734
	60–70	—	—	−769

Table VI. The optimal program

Process number	Name of products	The optimal production level *a*	*b*	*c*
		yards		
2	W_2	240,000		
7	M_2	60,000		
9	M_3	100,000		
11	M_4	305,190		
17	M_6	50,000	=*a*	=*a* or *b*
22	M_8	48,000		
24	M_9	20,000		
26	M_{10}	24,810		
27		5,190		
28	M_{11}	20,000		
		pounds		
33	S_1	530,110	554,330	
35	S_2	460,720	436,500	
38	S_4	70,000	70,000	40,200
39		0	0	29,700

Maximum profit: ¥. −663,789.

However, there still remain much to be investigated. This solution is not a final one for the firm. We must add the third limitation, "political limitation", which has rather non-economical feature, to the above two kinds of limitation, technological and market ones. For instance, the above solution gives a relatively small order to the subconstracted mills. From the political point of view, the firm can not reduce its amount of order to such a low extent, for the sake of future possibilities. Also, there are some products which require a constant level of production for the maintenance of brands. These requirements can and should be added to the above programming only when it has been solved.

THE MAXIMUM EXPANSION OF BANK CREDIT AND THE THEORY OF GAMES

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The greater part of this paper was read at Econometric Seminar, Princeton University on Feb. 3 last year, in compliance with kind invitation of Professor Oskar Morgenstern, Princeton University. Availing myself of this opportunity, I would like to express my deep gratitude to Professor Oskar Morgenstern, under whose kind guidance I could study recent developments of the theory of games at Princeton University.

Beginning with Phillips, many writers such as Lawrence, Angell, Shaw[1] and others have written upon the maximum expansion of bank credit. Among others, Lawrence's article, "Borrowed Reserves and Bank Expansion," which appeared in the Quarterly Journal of Economics, August, 1928 seems to have been one of the best known. Lawrence's description is not clear as to how he could get the figure of the final coefficient of expansion, namely 5.23.[2] The process of calculation to get the figure was not made explicit. One can not decide whether his result is correct or not from his description. And yet, strangely enough, it was left unquestioned or at least unsolved until 20 years later, namely, until 1949. At that time Professor Tanaka took up this problem[3] and questioned its validity, calculation and derivation of the figure, and what a formula of calculation we should use in order to get right coefficient.[4] The author collaborated with Professor Tanaka to solve this

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[1] C. A. Phillips, *Bank Credit*, New York, 1923.
J. S. Lawrence, "Borrowed Reserves and Bank Expansion" *The Quarterly Journal of Economics*, August, 1923.
J. S. Lawrence, *Stabilization of Prices*, New york, 1928.
J. W. Angell and K. F. Ficek, "The Expansion of Bank Credit, I," *The Journal of Political Economy*, Feb., 1933.
E. S. Shaw, *Money, Income and Monetary Policy*, Cicago, 1950.

[2] J. S. Lawrence, *Stabilization of Prices*, New York, 1928, p. 367.

[3] Pofessor Kinji Tanaka, professor of the Faculty of Business Administration, Kobe University, later published, "Coefficient of Expansion of Bank Credit" in *Essays in Commemoration of the 50th Anniversary of the Foundation of Kobe University of Economics*, Kobe, 1953.

[4] Professor Tanaka pointed out at the same time that the secondary and the further coefficients of expansion must have been Phillip's instead of Lawrence's e=1.79219.

problem and we arrived at a solution. It was found that Lawrence's statement that the final coefficient of expansion is 5.23 and that the other banks playing entirely passive roles have expanded 13.306 per cent (which must correctly be 7.5984 and 6.3675×5.5792 respectively under Lawrence's assumptions) are completely wrong and cannot possibly have any logical foundation. During the summer of 1949, Professor Tanaka lectured on this subject at a meeting for the study of monetary theory. The author discussed the discrepancy under the title, "A New Form of Multiplier" in a Tokyo Meeting of the Japan Society of Economics in December, 1949. The discussion was published in a Japanese Journal which appeared in January, 1950.[5]

Under similar assumptions such as Lawrence has taken, let us take a self-sufficient community whose banking facilities are provided by n Banks $B_1, B_2, \ldots, B_n$. Let us assume further that the coefficient of loan expansion of B_i $(i=1, \ldots, n)$ be e_i in each period and that of this amount e_i of the loan expansion Bank B_i, the amount of $e_i q_{ik}$ will flow into the Bank B_k to be the deposit of that Bank in the next period (q_{ik} being, of course, a proper fraction).

Now suppose that in the period "0" there were additional increases of deposits in each of all n Banks, the additional deposit of Band B_j being $\bar{D}_j = f_j(0)$ $(j=1, \ldots, n)$ and that all banks expand their credits each according to their coefficient of loan expansion, e_j, for example, for the Bank B_j $(j=1, \ldots, n)$.

Thus, in period "t", for example, we assume the additional deposit of the Band B_i caused by these and subsequent loan expansions of all n banks to be $f_i(t)$. According to these assumptions, we have the following system of difference equations:

$$f_k(t) = \sum_{k=1}^{n} r_{ik} f_i(t-1) \qquad [k = 1, \ldots, n]$$

where $r_{ik} = e_i q_{ik}$ and $r_{ii} = 0$, whose initial conditions are

[5] Kazuo Midutani, "A new Form of Multiplier", *Riron-Keizai-Gaku*, Vol. 1, No. 1, Tokyo, January 1950. The first part of the following is the purport of what was published in that journal. It contains a system of simultaneous difference equations similar to that which Dr. Chipman used in his book, "*Theory of Inter-Sectoral Money Flows and Income Formation*", Baltimore, 1951, but it must be noticed that the treatment of it is fundamentally different from that of Dr. Chipman.

$$f_i(0) = \bar{D}_i \qquad [i = 1, \dots, n]$$

Let the roots different with each other of the characteristic equation of the above system of difference equation:

$$\varphi(\lambda) = |r_{ij} - \delta_{ij}\lambda| = 0 \qquad [i, j = 1, \dots, n],$$

where r_{ii} being 0, be $\lambda_1, \lambda_2, \dots, \lambda_s$ and the multiplicity of λ_l be m_l $[l = 1, \dots, s]$.

Then the general solution of the above system of difference equation is expressed as

$$f_k(t) = \sum_{l=1}^{s} \lambda_l^t \left\{ \sum_{j=1}^{m_l} A_{jlk} \binom{t}{m_l - j} \right\}$$

where $\binom{t}{m_l - j}$'s are binomial coefficients, and A_{jlk}'s are determined by the given initial conditions as usual. But this is only formally possible and it is generally impossible to express λ's applying four rules of arithmetics upon r_{ij}'s and extracting hth roots of them (h being any positive integer).

However, so long as numerical values of λ's are less than unity,[6] the total sum of $f_k(t)$'s over an indefinitely long period (t runs from 0 to infinity) is finite and definite. Let us designate this total sum of $f_k(t)$'s as D_k, that is

$$D_k = \sum_{t=0}^{\infty} f_k(t) \qquad [k = 1, \dots, n]$$

By summing up each of the difference equations with respect to t from 1 to ∞, we have

$$D_k - \bar{D}_k = \sum_{i=1}^{n} r_{ik} D_i$$

From this system of simultaneous linear equations, it is easy to get D_k's as usual, and consequently L_k's as functions in D_k's, r_{ik}'s and e_k's $[i, k = 1, \dots, n]$, because L_k's are each equal to $e_k D_k$'s respectively, where L_k signifies the ultimate maximum amount of expansion of credit of Bank B_k caused by additional increases of deposits $\bar{D}_1, \bar{D}_2, \dots, \bar{D}_n$ in each of all n banks respectively in the period "0".

Now e_k's and consequently r_{ik}'s $[k = 1, \cdots, n]$ are functions in the

[6] The conditions for this are given by Cohn-Schur's Theorem.

reserve ratio's $\rho_1, \rho_2, \cdots, \rho_n$ of all n banks. Therefore, D_k's are functions in ρ_k's $[k=1, \cdots, n]$, so long as the all n banks' $\bar{D}_k$'s are given.

In order to apply the theory of games[7] to this problem, we must assume that the pay-off function should be the following one for Bank B_1:

$$H(\sigma_1, \sigma_2) = 2L_1 - (\sum_{j=1}^{n} L_j)$$

where L_k's and consequently H are all continuous functions in σ_1 and σ_2. Thus, σ_1 and σ_2 signify the strategies of Bank B_1 and the other $n-1$ banks $B_2, B_3, \cdots, B_n$ respectively and represent any point in compact subsets T_1 and T_2 of Euclidian space respectively. The dimension of T_1 is 1 and that of T_2 is $n-1$. This σ_1 represents ρ_1 the reserve ratio of Bank B_1 and coordinates $(\sigma_{22}, \sigma_{23}, \cdots, \sigma_{2n})$ of σ_2 represent $\rho_j(=\sigma_{2j})$'s the reserve ratio's of Bank B_j's respectively $(j=1, \cdots, n)$.

Further, let

$$G(\varphi_1, \varphi_2) = \int_{T_1}\int_{T_2} H(\sigma_1, \sigma_2)\, d\varphi_1(\sigma_1)\, d\varphi_2(\sigma_2)$$

where φ_i is any probability measure defined over T_i $[i=1, 2]$. Then, by the fundamental theorem of the zero-sum two-person continuous game, it is easy to see that there are functions $\varphi_1^0(\sigma_1)$ and $\varphi_2^0(\sigma_2)$ such that[8]

$$\max_{\varphi_1 \in \Phi_1} G(\varphi_1, \varphi_2^0) = G(\varphi_1^0, \varphi_2^0) = \min_{\varphi_2 \in \Phi_2} G(\varphi_1^0, \varphi_2)$$

Thus, we can deduce from this that Bank B_1 should adjust its reserve ratio's according to their probability density φ_1 in order that the Bank B_1 may maximize its credit expansion on condition that the initial increases of bank deposits $\bar{D}_k$'s $(k=1, \cdots, n)$ should be given at the outset.

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[7] John von Neumann and Oskar Morgenstern, *The Theory of Games and Economic Behavior*, Princeton, 1944.

[8] The demonstration of the theorem in such a continuous case was given independently by Dr. L. S. Shapley and by Dr. Tetsuya Gonoh.